A Fast Finite Element Modelling Tool for Surgical Simulation

Naoufel AZOUZ
Centre d'Etudes de Mécanique d'I-D-F
Laboratoire des systèmes complexes
Université d'Evry Val d'Essonne
40 rue du Pelvoux, 91020 Evry, FRANCE.
E-mail : azouz@cemif.univ-evry.fr

Shahram PAYANDEH
Experimental Robotics Laboratory (ERL)
School of Engineering Science
Simon Fraser University
Burnaby, British Columbia, CANADA.
E-mail : shahram@cs.sfu.ca

ABSTRACT

This paper present an algorithm based on finite element modelling of 3D solid volumetric, suitable for real-time animation and for haptic interaction, particularly for surgical simulator. The techniques used now by most surgical simulators are based on approximate methods that do not represent an accurate behaviour of the deformable bodies. The finite element method (FEM) is an alternative because of its accuracy, but it is computationally expensive. In this paper we address this problem by proposing the notion of ε-mesh radius in analogy with the similar notion used in the theory of adaptive meshing.

Keywords : Modelling, Finite-Element, Surgical Simulation, ε-mesh radius.

1. INTRODUCTION

The surgical simulation becomes possible, through the improvement of the computers, and some special purpose mechanisms, namely haptic interface. Various graphical based environment are being identified where the user is able to interact with such virtual environment. In these environments, the user manipulates graphical objects by using the hand interface mechanism. If the interference of the graphical object is detected, desired reaction forces are generated at the hand input device which gives the user a sense of physical interaction with the virtual objects.

Current literature in the area of haptic rendering focuses on the development of fast collision detection algorithm and some basic contact profiling [1]. For modelling the interaction with the soft tissues different approaches have been investigated [2].

Some authors proposed modelling approaches based on the spring-mass method because of their simplicity and their low cost computational [3,4]. However, such current approaches remain to be imprecise. On the other hand, The finite element method (FEM) offers an alternative. The main problem in this method is the high computational time.

Cotin & al. [5] tried to overcome this problem by using a pre-processing technique and the condensation method. Berkley and Weghrost [6] proposed to use the technique of the banded matrix. They also proposed to incorporate the whole volume of the tissue and organs for generating the finite element mesh.

This paper is concerned with determining a suitable method for defining the contact profile for constrained objects. The objective of the paper is to define suitable size and number of local grid-mesh which can represent a suitable physical recreation model for the contact profile with realistic deformation. For this purpose a number of comparisons between the traditional generation of the finite-element mesh and a new notion of ε-mesh radius is made especially in the context of piercing and cutting human tissue. As a result this study offers an approach to reduce the computation time for determining the contact profile using the notion of finite-element. In addition, such new approach can be made adaptive where, as a function of interference parameters and location of the contact between bodies, local mesh can be generated. This mesh profile can be defined optimal for computational purposes in haptic rendering. When contact is broken with the virtual object, the object representation will have its original coarse representation of mesh defined through CAD environment (or generated through MRI data-sets). The prototypes of the analysis of this study are developed using general purpose finite element environment (CASTEM 2000).

2. Overview

2.1 Mathematical formulation

In this section We briefly introduce the theory of elasticity and the finite element method.
Let us consider a body (Ω) having a volume V and a boundary $(\partial\Omega)$. At first approach, we consider that the object has a linear elastic behaviour. It is subject to volumic and boundary forces.

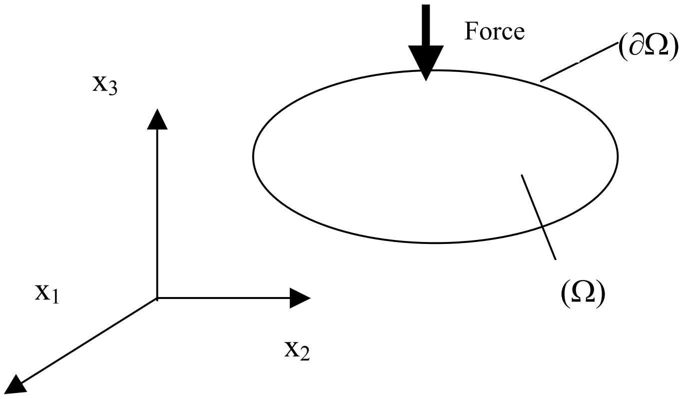


Figure 1. Description of elastic body

From the linear elasticity, the 3x3 strain tensor $\overline{\overline{\varepsilon}}$ of the body subject to a given force could be written as a function of the displacement **U** as :

$$\overline{\overline{\varepsilon}} = \frac{1}{2}\left(\nabla U + \nabla^{T} U\right) \qquad (1)$$

∇ is the gradient operator.
Let us note $\overline{\overline{\sigma}}$ the 3x3 symmetric stress tensor and f_v the volumic forces applied on the body . In the case of elastic behaviour, the relation between stress tensor and strain one is :

$$\overline{\overline{\sigma}} = \overline{\overline{E}}.\overline{\overline{\varepsilon}} \qquad (2)$$

$\overline{\overline{E}}$ is the four order tensor of the elasticity coefficients, it depends on the mechanical characteristics of the body (Ω).
When using the principle of virtual works, one can obtain, for a virtual increment of displacement δU :

$$-\int_V \bar{\sigma}.\delta\bar{\varepsilon}\, dV + \int_{d\Omega} t_n.\delta U\, dS + \int_V f_V.\delta U\, dV = 0 \qquad (3)$$

t_n is the stress vector applied on the boundary ($\partial\Omega$).
To overcome the complexity of modelling objects based on continuum mechanics, i.e. solving equation (3), the body (Ω) is now divided into different elementary domains limited by nodes. The displacement of a given point of the body could be approximate as a function of the nodal displacements as [7] :

$$U = [N]^{T}.\bar{U} \qquad (4)$$

[N] is the matrix of interpolation functions and $\bar{U}$ is the displacement column matrix of the nodes.
Noting the global stiffness matrix of the form :

$$[K] = \int_V (D[N])^{T}.E.D[N]\, dV \qquad (5)$$

D is a differential operator. The static equation (3) becomes for the discretized body :

$$[K].\bar{U} = \bar{F} \qquad (6)$$

$\bar{F}$ here represent the column matrix of the whole volumetric and boundary forces applied on the body.

2.2 Multigrid

In the minimally invasive surgery, that we propose to simulate, the region influenced by the action of the surgeon tool is localised in the vicinity of the contacting point. The reaction force induced by this movement is mainly depending on this limited zone. To avoid solving equation (6) with a great number of degrees of freedom, we divide the object into a number of volumes. In the neighbourhood of the contacting point, we generate fine mesh. We define the size of this local mesh by the ε-mesh radius which represent a characteristic length in this volume. For the remaining of the volume, we use coarse mesh. In classical FEM, this procedure should be accompanied by the minimisation of the errors [7] in the energy norm in relation with the validity of the FEM approximation.

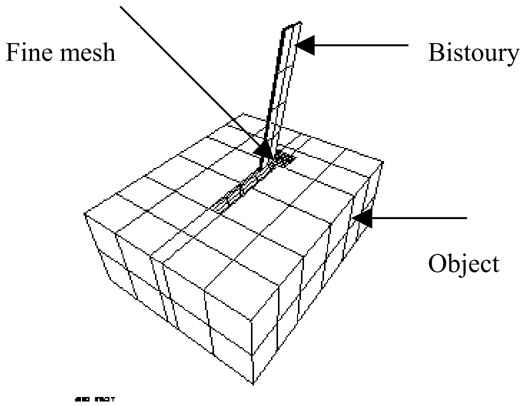


Figure 2. Modelling of the cutting problem

In haptic rendering, we have fewer constraints regarding the accuracy of the computation of the reaction force. In order to study this hypothesis, we will study through simulation the ratio between the reaction force given by our modelling method using ε-mesh and the one obtained through a full order

3D mesh representing the “exact” solution. The example selected correspond to the simulation of the cutting problem (figure 2).

3. Simulation results

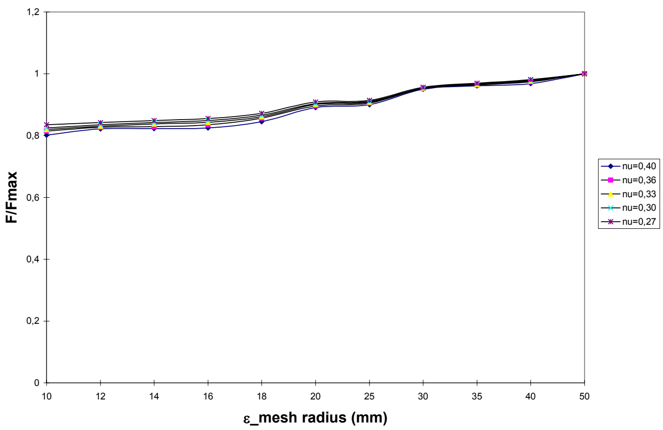
In order to determine the relationship between the full order 3D mesh and the ε–mesh radius technique, we impose a given displacement (of piercing or cutting) to the body and determine the force needed for such task. This in general is called inverse problem.

We purpose an algorithm where we can change different parameters in the simulations and compute the ratio between the “real” reaction force computed using the full-order finite-element model and the proposed ε–mesh procedure.

1. **Input** ← (geometrical shape, mechanical characteristics, types of mesh) ;
2. **Input** ← (locations of concentration, geometry of the probe);
3. **do** ←(calculate the real reaction force to an imposed displacement of piercing or cutting);
4. **do** ← (calculate the reaction force with a multigrid referenced by an ε–mesh radius);
5. **do** ← (recalculate the reaction force with a new ε–mesh radius till the global volume of the body);
6. **back to 1** (with new parameters);

end;

Figure 3 shows an example of our study. The plots represent the ratio between the reaction force given by the ε–mesh procedure and the one obtained by a



full order 3D mesh (F/F_{max}) for different values of the poisson ratio.

The figure shows that for a relatively small ε-mesh radius (10mm) , we obtain more than 80% of F_{max}. This ratio remain quasi-constant when we change different parameters of the study. Other simulations show that the ratio F/F_{max} could be considered as constant for different geometric and mechanical characteristics of an elastic body. Consequently we

Figure 3. Influence of poisson ratio

can develop a relationship between the results given by a full order 3D mesh issued from an off-line simulation, and those given by ε-mesh simulation. On the other hand, the computing time decreases significantly (about 50 times lesser) from the computation with a full order 3D mesh to the computation with the “ε-mesh radius” technique.

4. Conclusion

We present a method for the simulation of linear elastic deformable bodies in surgical applications. We proposed and are developing the new notion of ε-mesh radius, and have shown that it is a useful tool due its local interaction model, accuracy, and simplicity of implementation. Preliminary study shows that the results are close to those of a full order 3D mesh within a reasonable radius from the contact area.

The main contribution of the paper is to determine a guideline for developing physically based models of the object based on finite element method. The guideline suggest an approach for determining the size and number of elements involved for creating a realistic reaction force calculations which can be used in haptic rendering. The trade-offs are between the realistic sense of deformation and the computational costs.

5. References

[1] PAYANDEH and Al. “Haptic Rendering: Toward Modelling of the Deformable objects”, Proceedings of the ASME Symposium on Teleoperation and Haptic Interface for Remote and Virtual Environment, pp. 45-51,(2000).

[2] PAYANDEH S., and AZOUZ N. “ Finite elements, mass-spring-damper systems and haptic rendering” Accepted for publication in the CIRA 2001, T002, July 2001, Banff, Canada.

[3] *BAUMANN R. and GLAUSER D.* "Force Feedback for Virtual Reality Based Minimally Invasive Surgery Simulator", Medicine meets virtual reality, Vol. 4, San Diego, CA, 1996.

[4] *MESEURE P. and CHAILLOU C.* "Deformable Body Simulation with Adaptive Subdivision and Cuttings", Proc. of the WSCG’97, pp. 361-370, (1997).

[5] *COTIN S. and DELINGETTE H. and AYACHE N.* "Efficient Linear Elastic Models of Soft Tissue for real-time Surgery Simulation ", Institut National de Recherche Automatique (INRIA), Report 3510, Oct., 1998.

[6] *BERKLEY J. and WEGHORST S.* "Fast Finite Element Modelling in Surgical Simulation : The Banded Matrix Technique ", Technical report, HIT lab., University of Washington, Seattle, WA (1999).

[7] ZIENKIEWICZ O.C. and TAYLOR R.L.”The Finite element method”, Eds. McGRAW-HILL, 4th edition, (1999).