Contact Modelling for Real-Time Simulation in Telepresence Applications

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Abstract. The paper introduces the use of a time-stepping integration scheme for real-time simulations of multi-body systems with non-smooth contacts taking frictional contacts and inelastic impacts into account. Extended by a discretised local model its utilization in a contact model library, specifically designed for telepresence applications, is presented.

1 Introduction

Applications in telepresence and teleaction consist of two essential components, a human operator and a telerobot. The real system at the teleoperator side can be completely or partly replaced by a virtual one [5], e.g. for operator training. Then a simulation has to provide the operator side with visual *and haptic* data that otherwise would have to be measured – in real time. The dynamics of a multi-body system with contacts can be expressed as $\mathbf{M}(\mathbf{q};t)\ddot{\mathbf{q}} - \mathbf{h}(\mathbf{q};\dot{\mathbf{q}};t) = \mathbf{f}_C(\mathbf{q};\dot{\mathbf{q}})$ with $\mathbf{M}$ denoting the mass matrix, $\mathbf{q}$ the vector of generalised coordinates, $\dot{\mathbf{q}}$ the vector of generalised velocities and $\mathbf{h}$ the vector of all gyroscopic and external forces, not including contact forces. The contact forces are separated in the smooth vector function $\mathbf{f}_C(\mathbf{q};\dot{\mathbf{q}}) = \mathbf{f}_C(\mathbf{g}_N;\dot{\mathbf{g}}_N;\dot{\mathbf{g}}_T)$, explicitly depending on the kinematic quantities, i.e. on the vector of normal distances $\mathbf{g}_N(\mathbf{q};t)$ and the vector of tangential relative velocities $\dot{\mathbf{g}}_T(\mathbf{q};\dot{\mathbf{q}};t)$. Such formulation can be insufficient if contacts between solids with high contact stiffnesses have to be considered.

2 Contact Modelling

Usual approaches such as penalty methods or the regularization of the friction characteristic lead to well-known problems of stiff differential equations requiring small integration steps. Due to the resulting computing time real-time applications can become impossible. Fortunately, in many situations the knowledge of the local contact behaviour is not of interest. Rather, the influence of local contact quantities can be neglected and the global behaviour of the subsystem has to be determined. A rigid body contact modelling with Coulomb friction can be illustrated as Signorini impenetrability condition [7] and friction cone (Fig. 1 a,b). As no contact stiffness has to be specified the else associated model uncertainty is circumvented. Replacing the smooth contact force $\mathbf{f}_C$ by the non-smooth normal and tangential contact forces $\boldsymbol{\lambda}_N = \mathbf{h}_{,N}\boldsymbol{\lambda}$, $\boldsymbol{\lambda}_T = \mathbf{h}\lambda_T\boldsymbol{\lambda}$ yields

$$\mathbf{M}(\mathbf{q};t)\ddot{\mathbf{q}} - \mathbf{h}(\mathbf{q};\dot{\mathbf{q}};t) - \mathbf{W}_N(\mathbf{q};t)\boldsymbol{\lambda}_N - \mathbf{W}_T(\mathbf{q};t)\boldsymbol{\lambda}_T = \mathbf{0}: \qquad (1)$$

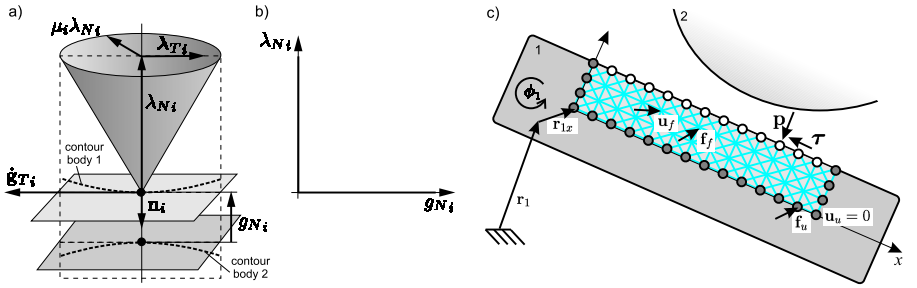

Fig. 1. Contact modelling: a) Coulomb friction cone and b) normal contact law, c) local embedded, discretised contact

The Jacobians $W_N = \partial g_N/\partial q$; $W_T = \partial g_T/\partial q$ span the space of normal and tangential contact forces. The contact law in normal direction can be written as complementarity condition: $g_N \geq 0$; $\lambda_N \geq 0$; $g_N^T \lambda_N = 0$: For Coulomb friction with a friction coefficient μ_i the contact condition can be given as the states for *slipping* $\lambda_{T\,i} = -\mu_i \lambda_{N\,i} g_{T\,i}/|g_{T\,i}|$ and for *sticking* $|\lambda_T| < \mu_i \lambda_{N\,i}$:

Due to the unknown contact forces λ_N and λ_T eq. (1) cannot be integrated directly by traditionally used schemes. Online simulation algorithms like those in [7] permit step size control but are only conditionally suitable for real-time applications: the time needed to compute the reduced time step can can easily exceed the simulated time Δt. Therefore a Time-Stepping integration scheme [3, 8, 9] with constant step size is applied. Its suitability has already been proven in many online applications, e.g. [2, 9]. Starting from eq. (1) a discretisation of $\dot{q}$ for a time step Δt is performed:

$$M(q;t)\Delta\dot{q} - h(q;\dot{q};t)\Delta t - W_N(q;t)\Lambda_N - W_T(q;t)\Lambda_T = 0: \qquad (2)$$

The contact forces have been replaced by impulses within Δt: $\Lambda_N = \lambda_N \Delta t$; $\Lambda_T = \lambda_T \Delta t$: With the Jacobian $\hat{W}_T = \partial g_T/\partial q$ and the kinematic excitations $w_N = \partial g_N/\partial t$, $\hat{w}_T = \partial g_T/\partial t$ the discretised normal distances and relative tangential velocities can be formulated as

$$g_N^{l+1} = W_N^T(q^{l+1} - q^l) + w_N \Delta t + g_N^l \qquad (3)$$

$$g_T^{l+1} = W_T^T(q^{l+1} - q^l) + \hat{W}_T^T(q^{l+1} - q^l) + w_T \Delta t + g_T^l \qquad (4)$$

with l and $l+1$ denoting the current and the next integration step. In order to formulate the normal contact law on position level the position has to be discretised implicitly: $\Delta q = q^{l+1}\Delta t$. As the solver requires a complementarity form for both, the normal and tangential contact conditions, the following separation of impulses and relative tangential velocities is necessary: $\Lambda_{T0}^{(+)} = \mu\Lambda_N + \Lambda_T \geq 0$, $\Lambda_{T0}^{(-)} = \mu\Lambda_N - \Lambda_T \geq 0$, $g_{T\,i}{}^{+} = \frac{1}{2}(|g_{T\,i}| + g_{T\,i}) \geq 0$, $g_{T\,i}{}^{-} = \frac{1}{2}(|g_{T\,i}| - g_{T\,i}) \geq 0$. The quantities $\Lambda_{T0}^{(+)}$ and $\Lambda_{T0}^{(-)}$ denote impulses of friction surplus in positive and negative direction, $\mu = \mathrm{diag}(\mu_i)$. In contrast $g_{T\,i}{}^{+}$ and $g_{T\,i}{}^{-}$ are positive and negative parts of $g_{T\,i}$. This separation is valid for planar problems only. For spatial problems an equivalent formulation can be found by transforming the friction cone into a N-sided pyramid, see [1, 3, 8].

The given equations result in the Linear Complementarity Problem (LCP)

$$\begin{pmatrix} g_N^{l+1} \\ \not{t} g_T^{+\,l+1} \\ \not{t} \alpha_{T0}^{(-)} \end{pmatrix} = \begin{pmatrix} G_{NN} & G_{NT}\mu & G_{NT} & 0 \\ G_{TN} & G_{TT}\mu & G_{TT} & E \\ 2\mu & & -E & 0 \end{pmatrix} \begin{pmatrix} \not{t} \alpha_N \\ \not{t} \alpha_{T0}^{(+)} \\ \not{t} g_T^{-\,l+1} \end{pmatrix}$$

$$+ \begin{pmatrix} W_N^T \not{t}(q^l + M^{-1} h \not{t}) + \hat{w}_N \not{t} + g_N^l \\ \left(G_T h + \hat{W}_T^T q^l + \hat{w}_T\right) \not{t}^2 + g_T^l \not{t} \\ 0 \end{pmatrix}; \quad \begin{pmatrix} g_N^{l+1} \\ \not{t} g_T^{+\,l+1} \\ \not{t} \alpha_{T0}^{(-)} \end{pmatrix}^T \begin{pmatrix} \not{t} \alpha_N \\ \not{t} \alpha_{T0}^{(+)} \\ \not{t} g_T^{-\,l+1} \end{pmatrix} = 0$$

$$\left(g_N^{l+1} \not{t} g_T^{+\,l+1} \not{t} \alpha_{T0}^{(-)}\right)^T \geq 0; \quad \left(\not{t} \alpha_N \not{t} \alpha_{T0}^{(+)} \not{t} g_T^{-\,l+1}\right)^T \geq 0 \qquad (5)$$

with $G_{NN} = W_N^T M^{-1} W_N$, $G_{NT} = W_N^T M^{-1} W_T$, $G_{TN} = G_T W_N$, $G_{TT} = G_T W_T$ and $G_T = (W_T + \hat{W}_T \not{t})^T M^{-1}$: As it is in standard form $y = Ax+b; x \geq 0; y \geq 0; x^T y = 0$ it can be solved by Lemke's algorithm [6]. Based on this solution the multi-body system can be updated to $t + \not{t}$. The step size, chosen small for a stable time-integration, would be disadvantageous for offline-simulations. However, for real-time simulations they have several advantages: First, less contact states changes per time step. Thus the solution algorithm can use the previous state for a faster determination of a valid current state. Second, the response time to any external initiated system changes is small. As the algorithm enforces the satisfaction of any normal constraint after each time step it is equivalent to an inelastic impact law, which is sufficient in many applications. So the time-stepping algorithm permits an efficient way to simulate mechanical systems with rigid contacts, friction and inelastic impacts [4].

On the other hand, if one is interested in the local deformation and contact force distribution, discretised contact models, Fig. 1c, can be used. For the computation of the contact forces p again a LCP has to be solved. With the solution of the quasi-static Finite Elements problem

$$\begin{bmatrix} K_{ff} & K_{uf} \\ K_{fu} & K_{uu} \end{bmatrix} \begin{bmatrix} u_f \\ u_u \end{bmatrix} + \begin{bmatrix} C_{nf} \\ C_{nu} \end{bmatrix} p + \begin{bmatrix} C_{tf} \\ C_{tu} \end{bmatrix} \tau - \begin{bmatrix} f_f \\ f_u \end{bmatrix} = 0 \qquad (6)$$

with $\tau = \tau(p; g_T)$ as a regularized friction force function, the multi-body system $M(q;t)\ddot{q}(t) - h(q;\dot{q};t) - W_u(q) f_u = 0$ can be updated. As these steps have to be performed in every time step efficient numerical methods are essential.

3 Contact Model Library

In order to bind these models and algorithms into telerobotic applications a contact model library is proposed. Due to the interdependence of contact modelling and integration algorithm the library cannot be solely limited to the contacts. Rather, the library represents a framework for implementing multi-body systems and their simulation in real-time. The library comprises two trees of C++ base classes of multi-body (sub)systems, and their linkage (Fig. 2). The system classes contain methods for the time-integration as well as for the data management of linked subordinated systems. Their hierarchical order corresponds to the combination of model types they can handle. The interfaces for time-integration and